Public Funding of Higher Education

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Abstract

Recent criticism from different sides has expressed the view that, with scarce resources, there is little justification for massive public funding of higher education. Central to the debate is the conjecture that colleges and universities use their resources inefficiently and focus insufficiently on their mission to expand students’ human potential. Our aim in this paper is to examine the theoretical premises of this conjecture in a small open economy and uncover the conditions under which public investment in higher education is efficient and desirable. We analyze non-stationary equilibria of an OLG economy, characterized by perfect capital mobility, intergenerational transfers and a hierarchical education system. The government uses income tax revenues to finance basic education and support higher education that generates skilled labor. Given this, the following issues are considered: (a) the impact of education and international markets on the equilibrium number of low-skilled and skilled workers in each generation; (b) the economic efficiency of public subsidies to higher education in generating skilled human capital; (c) the endogenous support for a government’s educational policies found in a political equilibrium.
1. Introduction

Higher education is currently being criticized by scholars, politicians, and the popular press who demand that higher education institutions undertake reforms. The claim is that colleges and universities bear the financial costs of very costly bureaucracies and other non-academic activities while in many cases fail to achieve their core mission of increasing the skills and human potential of the individual student (see Hacker and Dreifus, 2010). These demands for value from higher education institutions have been triggered by ever rising tuition fees and shaky economic conditions. This is happening worldwide but is more pronounced in Western countries where governments plan to cut their contributions to higher education (see, e.g., UK, USA, the Netherlands and Israel). Since public resources are generally scarce, choices have to be made and the following questions are often raised: (i) What is the justification for public participation in funding higher education? (ii) For developing countries, should funding of higher education be a priority or, perhaps, should resources be used to upgrade the quality of compulsory schooling? The objective of this paper is to address these tradeoffs formally in an open-economy equilibrium framework.

Nowadays, educational policy can hardly be implemented without incorporating some relevant international aspects, even for decisions that are considered 'domestic' such as basic schooling. In most countries, especially for the developed ones, higher education generates a significant part of a country’s stock of skilled labor. As a result, it affects the marginal returns to physical capital and channels the limited supply of foreign investments. Despite its importance there are very few studies that capture the way in which international market conditions directly influence governments’ allocation of resources and individuals’ decision-making regarding the acquisition of additional training and skills.
Balancing the government budget is an important constraint on education policy. This has been expressed by the popular view:

“If you want to have a new program, figure out a way to pay for it without raising taxes” US Senate Majority Leader H Reid.

This quote stresses the importance of including both sides of the government balance sheet when the effects of new policies are examined. This issue is also confirmed by studies dealing with the empirics of growth which show that the growth effects of public education spending are generally mixed except when the method of finance is properly accounted for in which case they are clearly positive (see, e.g., Bassanini and Scarpenta, 2001; Blankenau et al., 2007b).

Lastly, another important point is the net social benefits that accrue from public investments in higher education. The social costs of acquiring skills include expenses incurred by society that performs the education and training, necessary expenses by each individual to acquire skills, as well as the foregone income that would have been earned otherwise. Low-skilled workers are important contributors to the government budget since the tax revenues collected from their labor income are used to finance all parts of public education, though they do not directly benefit from these investments (see Garrat and Marshall, 1994; Fernandez and Rogerson, 1995; Gradstein and Justman, 1995; Bevia and Iturbe-Ormaetxe, 2002). The social benefits include higher earnings enjoyed directly by individuals as well as the indirect benefits that the economy derives from the human capital generated via the higher education system. The latter include, for example, a capacity to absorb new production technologies, a higher marginal return to physical capital which gives rise to inflows of foreign physical capital. Given this background, is a government funding policy, like a subsidy to all individuals who wish to attend higher education, going to lead to
a net social benefit? Other programs like poverty relief and improved basic education may generate a higher social value than investing in higher education (Johnson, 1984). This paper studies 'efficient' education policies in small open economies.

Our analysis is carried out in an overlapping-generations model with heterogeneous agents and, starting from some initial conditions, computes and traces non-stationary competitive equilibria. Parents are altruistic in that they care about their offspring and derive utility from his/her lifetime income. Within this setting, the following issues will be analyzed in equilibrium: (i) the partition of the set of individuals between low-skilled and skilled workers in each generation; (ii) the evolving role of public subsidies to higher education on efficiency and the stock of human capital; (iii) the endogenous support for government educational policies generated within a political equilibrium.

Using a general process of hierarchical education and comparing dynamic equilibrium paths period by period, we obtain the following results: (a) Under certain conditions some public support in funding higher education will enhance the economy’s stock of human capital and growth; (b) Under certain conditions, society may be better off when no public funds are allocated to higher education; (c) The shape of the distribution of endowments of individuals matters for the allocation of public funds in a political equilibrium. In a society with a majority of low-skilled workers the median voter will oppose any public financing of higher education; (d) In equilibrium with a balanced budget, the marginal rate of substitution between expenditure on basic education vs. higher education is larger than unity; (e) If an open economy is relatively more endowed with physical capital, then upon free capital mobility, outflows of physical capital will increase the unskilled labor force.
Some features of our model have been analyzed before in other hierarchical education frameworks. Particularly, Driskill and Horowitz (2002) study the optimal investment in hierarchical human capital and find that the optimal program exhibits a non-monotonicity in human capital stocks. In Su (2004) the emphasis is on efficiency and income inequality in a hierarchical education system. She also studies the effects on growth of introducing subsidies to higher education (while total education budget assigned to basic and higher education is fixed). Su (2006) examines the endogenous allocation of the public budget when a top class has a dominant political power. Blankenau (2005) finds a critical level of expenditure above which higher education should be subsidized since its impact on growth is positive. Arcalean and Schiopu (2010) study the interaction between public and private spending in a two-stage education system. As in our framework, they observe that increased enrolment in tertiary education does not always enhance economic growth.

The paper is organized as follows. Section 2 outlines the individual preferences, describes the multistage formation of human capital in an OLG economy and characterizes the non-stationary competitive equilibria. Section 3 studies the partition of the workforce into 'low-skilled' and 'skilled' workers and its dependency on education variables and international factor prices. Section 4 analyzes the implications of public funding of higher education for growth and for efficiency. Section 5 introduces a political equilibrium in our model and examines majority voting to allocate education tax revenues. Section 6 contains concluding remarks. The Appendix contains most of the proofs to facilitate the reading.

2. The Economic Framework and Dynamic Equilibrium

Our research strategy in this section is first to specify the lifetime preferences of agents and derive their optimal behaviour. Optimal decision variables are then
aggregated to obtain variables like the economy’s human capital and government budget balance. Subsequently, the competitive equilibrium is fully characterized.

**Preferences and Hierarchical Education**

Consider an overlapping generation economy with a continuum of agents in each generation. Each agent is characterized by a family name \( \omega \in [0,1] \) where \( \Omega = [0,1] \) denotes the set of all families in each generation and \( \mu \) the Lebesgue measure on \( \Omega \). Each agent lives for three periods: a study period, a working period and a retirement period. During the early stage each child is engaged in education/training, but takes no economic decision like schooling, consumption or saving. Youth is followed by adulthood which is split in two periods: individuals are economically active during the working period and later enter the retirement period. Agents give birth to one offspring at the beginning of their working period such that population growth is zero. Hence, at any date \( t \), three generations with the same family name co-exist: (1) the child, born at the outset of date \( t \), who gets his education/skills; (2) the parent, born at date \( t-1 \), who takes economic decisions; (3) the grand-parent, born at date \( t-2 \), who consumes his savings.

Consider generation \( t \), denoted \( G_t \), consisting of all individuals born at the outset of date \( t \), and let \( h_{t,1}(\omega) \) be the human capital of family name \( \omega \) at the beginning of the working period. We assume that \( h_{t,1}(\omega) \) is achieved by a hierarchical production process of human capital like in Restuccia and Urrutia (2004): it consists of fundamental education (assumed to be compulsory) and \( ^2 \) higher education. A child obtains his general skills from the basic education and acquires eventually specialized skills from higher education. Innate ability of

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\(^2\)See also Su (2004), Blankenau and Camera (2006).
an individual $\omega \in G_i$, denoted by $\tilde{\theta}_i(\omega)$, is assumed to be random and drawn (at birth) from a time-independent distribution. Namely, we assume that abilities are independent and identically distributed random variables across individuals in each generation and over time.

The empirical literature has established that parental inputs together with school inputs are key factors affecting the human capital of individual $\omega$ while attending compulsory education. Both inputs are included in our process of human capital formation:

\[ h_{t+1}(\omega) = \tilde{\theta}_i(\omega)h_i^\nu(\omega)X_t^{\xi} \]

where $h_i(\omega)$ stands for parents’ human capital and $X_t$ represents public investment in early-life and compulsory schooling.\(^3\) The above human capital formation process is a representation of the complex interaction between innate ability, family dynamics and public intervention. It stresses the key role played by the individual home environment that is specific to each $\omega$ via the individual parental human capital and the public resources invested in public education that are common to all. The elasticities $\nu$ and $\xi$ represent the effectiveness of parents’ human capital in their efforts towards educating their child, and the efficiency of public education in generating human capital respectively: $\nu$ is affected by home education and family background while $\xi$ is affected by the schooling system, teachers, size of classes, facilities, neighborhood, etc.

Enrollment in higher education is costly and, in most countries, requires

\(^3\)Researchers in a number of fields have showed that investments in care and education early in children’s lives carry high individual and social rates of returns. The most recent evidence is reviewed in Cunha et al. (2006). It is therefore not surprising to see increases in pre-primary enrolments. In a number of OECD countries (The Czech Republic, Germany, New Zealand and Poland) annual expenditures per student are higher on pre-primary education than on primary education (OECD, 2009, Table B1.1a).
the payment of a tuition fee at each date $t$, denoted $z_t^*$ and assumed to satisfy: $z_t^* > 1$. We assume that the government may participate in the cost of higher education and finance these subsidies by taxing the wage income of working individuals. Denote by $g_t$ the government (or public) allocation to each student wishing to attain additional skills via the higher education system. Thus, $z_t(\omega) = z_t = z_t^* - g_t$ is the net payment that each individual pays at date $t$ to access higher education.\(^4\) The cost of higher education is thus the same for all students of the same generation. For simplicity, we assume that the tuition and public funding are denominated in dollars of the working period of the student (e.g., it can be financed by students loan) and, throughout our analysis, we take the education tax imposed on wage incomes to be constant at the rate $\tau$.

We assume that acquiring higher education augments each individual’s basic skills by some factor $B > 1$. Thus if agent $\omega$ invests money $z_t^*$ and time, then his/her human capital accumulates to the level:

\[
(2) \quad h_{t+1}(\omega) = Bh_{t+1} = B\tilde{\theta}(\omega)h_t^*(\omega)X_t^\xi
\]

He/she is then called a **skilled worker** (denoted by $s$). To simplify our analysis (without restricting the generality) we assume that $B$ is time-independent. In contrast, if an agent $\omega \in G_t$ does not enroll in higher education, his/her human capital is determined solely by compulsory schooling education:

\[
(3) \quad h_{t+1}^l(\omega) = h_{t+1}(\omega) = \tilde{\theta}(\omega)h_t^*(\omega)X_t^\xi
\]

We call this agent a **low-skilled worker** (denoted by $l$). Instead of attending

\(^4\)Public funding provides only a share of investments in tertiary education. In 2006 the proportion of private funding of tertiary education ranged between 3.6% in Denmark and 83.9% in Chile (OECD, 2009, Table B3.2b). Different combinations of tuition fees and government subsidies in our model can reproduce the relative importance of private funding observed in the data.
some higher education institute after his basic education is attained, a low-skilled agent works during part of his youth using the basic skills given in (3). We assume that all low-skilled individuals do work during a portion $m$ ($0 \leq m < 1$) of their youth period. Since they work fully at period $t+1$ as well, the lifetime after-tax wage income earned by a low-skilled worker $\omega$ is:

\[
(1-\tau)h'_{t+1}(\omega)\left[mw_t(1+r_{t+1})+w_{t+1}\right]
\]

where $(1+r_{t+1})$ is the return to capital at date $t+1$; $w_t$ and $w_{t+1}$ are the wage rates per unit of effective labor at date $t$ and $t+1$ respectively. In contrast, a skilled worker’s after-tax lifetime wage earnings derive from performing work only during period $(t+1)$:

\[
(1-\tau)h'_{t+1}(\omega)w_{t+1}
\]

There is little disagreement about the presence of intergenerational transfers (between parents and their children) in developed and developing countries. These transfers arise from altruistic motives of parents, regarding the well-being of their child, and are expressed in the various forms of investment in education that affect future earnings, and of tangible transfers like \textit{inter vivos} gifts and bequests (see Viaene and Zilcha, 2002; Zilcha, 2003). In our framework, we assume that parents care about the future of their offspring and derive utility directly from the lifetime income of their child.\footnote{Thus we depart from the dynastic model where the utility functions of all future generations enter this utility function.} We take the lifetime preferences of each $\omega \in G_t$ to be represented by the Cobb-Douglas utility function:

\[
U_t(\omega) = \left(c^a_t(\omega)^{\alpha_a} \left(c^r_t(\omega)\right)^{\alpha_r} \left(y_{t+1}(\omega)\right)^{\alpha_y}\right)
\]

Consumption when 'active' and 'retired' are denoted by $c^a_t(\omega)$ and $c^r_t(\omega)$ respectively;
$y_{t+1}(\omega)$ is the offspring’s lifetime income. Intergenerational transfers that arise from the altruistic motives represented by (A1), take three forms. First, the earning capacity of the younger generation is enhanced by taxes paid by parents to finance the public education budget, and as a result to enhance their human capital. Second, parents are willing to contribute to the tuition fees that allow access to higher education. Lastly, under the above preferences, parents transfer tangible assets directly as well.

Denote by $h_t(\omega)$ the transfer of physical capital by household $\omega \in G_t$ to his/her offspring. Given the return to capital and wages $\{r_t, w_t\}$, lifetime non-wage income of an offspring, whether skilled and low-skilled, is $(1 + r_{t+1})b_t(\omega)$. Thus, lifetime income of a low-skilled worker is:

(5) $y'_{t+1}(\omega) = (1 - \tau)h'_{t+1}(\omega)[mw_t(1 + r_{t+1}) + w_{t+1}] + (1 + r_{t+1})b'_t(\omega)$

In contrast, if he/she is a skilled worker then:

(6) $y'_{t+1}(\omega) = (1 - \tau)h'_{t+1}(\omega)w_{t+1} + (1 + r_{t+1})b'_t(\omega)$

Given (2) and (3) it is straightforward to obtain the aggregate (or mean as well in our case) human capital $H_t$ that is available to the economy at date $t$. Let $A_t$ denote the subset of individuals in $G_t$ who are skilled and let $\sim A_t$ be the complement of $A_t$, namely the set of low-skilled individuals. Hence:

(7) $H_t = \int h_t(\omega)d\mu(\omega) + m \int h'_{t+1}(\omega)d\mu(\omega)$

Therefore, government tax revenues are simply $\tau w_t H_t$ where $H_t$ is defined in (7). On the other side of its balance sheet the government faces total education expenditure (in both stages). Denote by $\mu(A_t)$ the measure of skilled individuals who receive some public funding for higher education. Then the government budget at date $t$
is balanced if the following identity holds:

\[
(8) \quad \tau w_t \left[ \int_{A_t} h_i(\omega) d\mu(\omega) + m \int_{A_t} h'_i(\omega) d\mu(\omega) \right] = X_t + g_t \mu(A_t)
\]

We say that an education policy \{(X_t, g_t)\} is feasible if at each date \(t\): (a) given \(X_t\) and \(g_t\), the set \(A_t\) of skilled agents is determined by each individual's 'optimal choice' and (b) condition (8) holds in all periods \(t\).

Define \(Z_{t+1}(\omega) = \tilde{\theta}_i(\omega) h_i(\omega)\) and call it the initial endowment of \(\omega\). It is the product of both ability and parental human capital and, hence, describes the background a young individual inherits prior to any education. Empirically it has been demonstrated that both factors are essential parts in the formation of offsprings’ human capital. For a fixed \(Z_{t+1}(\omega)\), there is a convex iso-endowment locus which connects all alternative combinations of \(\tilde{\theta}_i(\omega)\) and \(h_i(\omega)\) (with marginal rate of substitution \(-\nu / h_i(\omega)\)) and endows learning children with this given level of endowment. Thus there is an iso-endowment map representing each level of \(Z_{t+1}(\omega)\).

In our framework this 'initial endowment' is important because it is the main tool by which the decision to attend tertiary education is made. In general, the distribution function of \(Z_{t+1}(\omega)\) over the continuum of agents has a complex derivation from the underlying variables. However, under our assumptions, the random ability is a time-independent i.i.d. process and, given the human capital distribution of the older generation, it is possible to derive the distribution \(Z_{t+1}(\omega)\).\(^6\)

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\(^6\) \(Z_{t+1}(\omega) = \tilde{\theta}_i(\omega) h_i(\omega)\) is formed as the product of two distributions whose algebra is explained in Springer (1979). Very likely, \(h_i(\omega)\) is log-normally distributed. Whether \(\tilde{\theta}_i\) has a uniform distribution or a log-normal distribution, the product \(\tilde{\theta}_i(\omega) h_i(\omega)\) is log-normal. However, the probability distribution function of \(Z_{t+1}(\omega) = \tilde{\theta}_i(\omega) h_i(\omega) h_i(\omega)^{\nu - 1}\) becomes unknown except for extreme values of \(\nu\). In all cases, it can be evaluated by implementing numerical algorithms as in Glen et al. (2004).
Production is carried out by competitive firms that produce a single commodity which is both consumed and used as production input. Physical capital \( K_t \) (assumed to fully depreciate) and effective human capital \( H_t \) (computed in (7)) are inputs of a neo-classical production function that exhibits constant returns to scale; it is strictly increasing and concave. We consider a small open economy that, as of date \( t = 0 \), is integrated into the rest of the world in two ways. First, the final good is freely traded which implies a single commodity price worldwide. Second, physical capital is assumed to be internationally mobile while labor is kept internationally immobile. Consequently, we expect physical capital to flow from the low-return to the high-return economy until its marginal product is equalized across regions. With the small economy assumption, \( \{r_t\} \) must be equal to the foreign interest rate.\(^7\) With similar final goods prices and equal interest rates, the domestic wage must equal the pre-determined foreign wage as long as production technologies are similar.\(^8\) Given this framework, any education policy that leads to human capital accumulation is expected to temporarily increase the domestic marginal return to physical capital and, hence, bring about an inflow of foreign physical capital. The increase in both primary inputs must increase domestic output.

**Competitive Equilibrium**

Given \( K_0, H_0 \), education policy \( \{(X_t,g_t)\}_{t=0}^{\infty} \), the international prices of

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\(^7\)There are several reasons why these returns may not be equalized. Barriers to capital mobility like capital controls and corporate income tax differentials would create a difference in rates of return. However, a less than full capital mobility would not modify our results qualitatively as long as the wedge in returns stays constant.

\(^8\)Particularly, wages are the solution to two iso-price equations of the model, one for each economy. With equal prices and interest rates, wages must be similar only when production technologies are the same in both economies. Different technologies would cause a cross-country difference in wages and trigger international migration. While physical capital is homogenous, human capital is not and this feature makes it difficult to determine the extent and the skill content of the labor flow.
capital and labor \( \{r_t, w_t\} \), and the tax rate \( \tau \), each agent \( \omega \) at time \( t \) with intergenerational transfers \( b_{t-1}(\omega) \) chooses the level of savings \( s_t(\omega) \) and bequest \( b_t(\omega) \) together with the financial investment in higher education \( z_t(\omega) \), so as to maximize:

\[
(9) \quad \text{MAX}_{s_t, b_t, z_t} [U_t(\omega) = \left( c^*_t(\omega) \right)^{\alpha_t} \left( c'_t(\omega) \right)^{\beta_t} \left( y_{t+1}(\omega) \right)^{\gamma_t}] 
\]

subject to constraints:

\[
(10) \quad z_t(\omega) = 0 \quad \text{or} \quad z_t(\omega) = z_t^* - g_t, \quad b_t(\omega) \geq 0 
\]

\[
(11) \quad c^*_t(\omega) = y_t(\omega) - s_t(\omega) - b_t(\omega) - z_t(\omega) \geq 0 
\]

\[
(12) \quad c'_t(\omega) = (1 + r_{t+1}) s_t(\omega) \geq 0 
\]

where \( y_t(\omega) \) and \( y_{t+1}(\omega) \) are the corresponding incomes given either by (5) or (6), while \( h^t(\omega) \) is defined by (2) for a skilled worker \( (z_t(\omega) = z_t^* - g_t) \) and \( h^t(\omega) \) is defined by (3) for a low-skilled worker \( (z_t(\omega) = 0) \).

Given \( K_0, H_0 \), \( \{(c^*_t(\omega), c'_t(\omega), s_t(\omega), b_t(\omega), z_t(\omega)); w_t, r_t\}_{t=0}^\infty \) is a \textbf{competitive equilibrium} if:

\( \textbf{(i)} \)\quad For each date \( t \), given factor prices \( (r_t, w_t) \) and the public education policy \( \{(X_t, g_t)\}_{t=0}^\infty \), the optimum under conditions (9)-(12) for household \( \omega \) with bequest \( b_{t-1}(\omega) \) is \( (c^*_t(\omega), c'_t(\omega), s_t(\omega), b_t(\omega), z_t(\omega)) \geq 0 \).

\( \textbf{(ii)} \)\quad Given the aggregate production function, the wage rate of effective labor \( w_t \) is determined by the marginal product of (effective) human capital.

\( \textbf{(iii)} \)\quad The education policy \( \{(X_t, g_t)\}_{t=0}^\infty \) is \textbf{feasible}, hence the government budget constraint in (8) holds at each date \( t \).
After substituting all constraints, first order conditions that lead to the necessary and sufficient conditions for an optimum are (assuming interior solutions):

\[
(13) \quad \frac{c_t^s(\omega)}{y_{t+1}(\omega)} = \frac{\alpha_1}{\alpha_3} \frac{1}{(1 + r_{t+1})} \quad \text{if } b_t(\omega) > 0 \text{ and }
\]

\[
(13') \quad \frac{c_t^s(\omega)}{y_{t+1}(\omega)} < \frac{\alpha_1}{\alpha_3} \frac{1}{(1 + r_{t+1})} \quad \text{if } b_t(\omega) = 0
\]

\[
(14) \quad \frac{c_t^s(\omega)}{c_t^f(\omega)} = \frac{\alpha_1}{\alpha_2} \frac{1}{(1 + r_{t+1})}
\]

We assume that intergenerational transfers are unidirectional and therefore cannot take negative values along the equilibrium path. This is guaranteed by the following sufficient condition:

Given the feasible education policy \( \{(X_t, g_t)\}_{t=0}^{\omega} \) and the international interest rate \( \{r_t\} \), then for all generations \( t \) and all \( \omega \in G_t \), the optimal consumption satisfies:

\[
(C) \quad \frac{c_t^s(\omega)}{y_{t+1}(\omega)} \geq \frac{\alpha_1}{\alpha_3} \frac{1}{(1 + r_{t+1})} \quad t = 1, 2, \ldots
\]

From (12), (13) and (14) we obtain that:

\[
(15) \quad y_{t+1}(\omega) = \frac{\alpha_1}{\alpha_2} (1 + r_{t+1}) s_t(\omega)
\]

Using (15) and the definitions of income in (5) and (6), we obtain the expression for bequest if the offspring turns out to become low skilled:

\[
(16) \quad b_t^l(\omega) = \frac{\alpha_1}{\alpha_2} s_t(\omega) - \frac{(1 - \tau)[mw_t(1 + r_{t+1}) + w_{t+1}]}{(1 + r_{t+1})} h_{t+1}^l(\omega) \geq 0
\]

Likewise for a skilled offspring:

\[
(17) \quad b_t^s(\omega) = \frac{\alpha_1}{\alpha_2} s_t(\omega) - \frac{(1 - \tau)w_{t+1}}{(1 + r_{t+1})} h_{t+1}^s(\omega) \geq 0
\]
Due to free capital mobility, both intergenerational transfers are affected by international market conditions. The reason is that when altruistic rational parents make forward-looking decisions regarding direct financial transfers and/or investment in attaining skills, they actually compare the return to physical capital with that of human capital. Thus, in such considerations they take into account the future interest rate and the future wage rate respectively.

Substituting (16) and (17) in (5) and (6) respectively, and making use of first order conditions (13) and (14) we obtain the reduced-form income of agent \( \omega \) who is either a low-skilled or a skilled offspring:

\[
\begin{align*}
\bar{y}_{t+1}^\ell(\omega) &= \left( \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \right) (1+r_{t+1}) \left( \frac{(1-r)(w_{t+1} + mw_t(1+r_{t+1}))}{(1+r_{t+1})} Z_{t+1}(\omega) X_t^\ell + y_t(\omega) \right) \\
\bar{y}_{t+1}^s(\omega) &= \left( \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \right) (1+r_{t+1}) \left( \frac{(1-r)w_{t+1}}{(1+r_{t+1})} B Z_{t+1}(\omega) X_t^s \right) - (z^* - g_t) + y_t(\omega)
\end{align*}
\]

These two expressions exhibit an intergenerational persistence of incomes that is:

\[
\frac{\partial \bar{y}_{t+1}^\ell(\omega)}{\partial y_t(\omega)} = \frac{\partial \bar{y}_{t+1}^s(\omega)}{\partial y_t(\omega)} = \left( \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \right) (1+r_{t+1})
\]

It is increasing in altruism parameter \( \alpha_3 \) and in the interest rate at the future date. Particularly, the persistence is similar for all households \( \omega \) since \( \alpha_3 \) is assumed to be the same for all families and \( (1+r_{t+1}) \) is given to all. In the next sections, both expressions for income will be crucial in partitioning the work force between skilled and low-skilled workers and determining agents’ political preferences.

3. Equilibrium Sets of Skilled and Low-Skilled Workers

The government budget sheet in (8) records among others the tax contributions made by low-skilled workers and the public subsidy that students in higher education receive while acquiring skills. Particularly, both student types are represented by \( A_t \).
and \( \mu(A_j) \). Hence, to be able to maintain government budget balance throughout our analysis, it is the important task of this section to determine both sets explicitly.

**Reduced-form Lifetime Preferences**

From the first order conditions (13) and (14) we obtain

\[
c_i(\omega) = \left(\frac{\alpha_1}{\alpha_2}\right) y_{t+1}(\omega) / (1 + r_{t+1}) \quad \text{and} \quad c_f(\omega) = \left(\frac{\alpha_2}{\alpha_3}\right) y_{t+1}(\omega) .
\]

After inserting these expressions into (9) the utility function has the following reduced form:

\[
U_t(\omega) = \Phi \left( \frac{1}{1 + r_{t+1}} \right)^{\alpha_1} \left[ y_{t+1}(\omega) \right]^{\alpha_2} + \alpha_3
\]

where parameter \( \Phi \) is a constant independent of time and independent of \( \omega \).

Therefore (18) is an expression for utility that holds for both skilled and low-skilled offspring. The reduced form utility of parents is now proportional to the lifetime income of their offspring where the term of proportionality is decreasing in the world interest factor at the future date. Thus, if education resources are allocated by a utilitarian social planner that maximizes the current aggregate of individual utilities, it maximizes at the same time next generation’s aggregate income. Also, whether parents invest in higher education of their child depends very much on their own utility, which entails comparison of future lifetime income of their child.

**Education Decision**

Making use of (18), the next result defines the proportion of the population that will receive higher education and become skilled. It sheds some light into the observed cross-country variations in the skill composition of workforces in both developed and developing countries. For example, Table 1 shows the skill composition of workforces for a subset of OECD countries and for OECD’s partner countries. The extent of a skilled workforce is approximated by the share of age group
25-64 with at least upper secondary education. Shares in 2007 vary largely, between 27 percent in Portugal and 89 percent in Estonia.

**Table 1: Cross-Country Variation of the Skilled Work Force**

<table>
<thead>
<tr>
<th>OECD Countries</th>
<th>Age Group 25-64 with at least Upper Secondary Education</th>
<th>Partner Countries</th>
<th>Age Group 25-64 with at least Upper Secondary Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>52</td>
<td>Brazil</td>
<td>37</td>
</tr>
<tr>
<td>Korea</td>
<td>78</td>
<td>Chile</td>
<td>50</td>
</tr>
<tr>
<td>Mexico</td>
<td>33</td>
<td>Estonia</td>
<td>89</td>
</tr>
<tr>
<td>Netherlands</td>
<td>73</td>
<td>Israel</td>
<td>80</td>
</tr>
<tr>
<td>Portugal</td>
<td>27</td>
<td>Russia Fed</td>
<td>88</td>
</tr>
<tr>
<td>Turkey</td>
<td>29</td>
<td>Slovenia</td>
<td>82</td>
</tr>
</tbody>
</table>

*Notes:* (a) The skilled workforce is approximated by the percentage of the population of age group 25-64 with at least upper secondary education; (b) In percentage, in 2007. Source: OECD (2009, Table A1.2A, column 1)

Given the distribution of \( Z_{t+1}(\omega) \) the next proposition defines the set of students who will attend higher education:

**Proposition 1:** Let \( A_t \) denotes the set of individuals who choose to invest in higher education at date \( t \). Then: (a) \( A_t \) is nonempty if and only if the following condition holds:

(19)
\[
\frac{w_{t+1}}{1+r_{t+1}} \geq \frac{m}{B-1} w_t
\]

(b) Assume that condition (19) holds. Define:

\[
\Lambda_t = \left( \frac{1}{1-\tau} \right) \left[ \frac{1}{(B-1)} w_{t+1} - mw_t \right] \left( \frac{1}{X^t} \right). \]

Then:

(20)
\[
A_t = \{ \omega \mid Z_{t+1}(\omega) \geq \Lambda_t \}
\]

Namely, all individuals with initial endowments above \( \Lambda_t \) become skilled workers.

We shall relegate all the proofs to the Appendix. \( \Lambda_t \) is a threshold that partitions the distribution function of \( Z_{t+1}(\omega) \). Assuming that condition (19) holds, all \( \omega \in G_t \) with an initial endowment above \( \Lambda_t \) will invest in higher education and
become skilled whereas the other individuals with an initial endowment below \( \Lambda \), will not invest and, hence, become unskilled.\(^9\)

### Comparative Statics

Some monotonicity results that can be verified from condition (20) are reported in Table 2 and should be interpreted as follows. Suppose that at date \( t \) an increase occurs in one of the model parameters of the first row, then the sign of the comparative statics of this change on either \( \Lambda_t \) or \( A_t \) is given in each relevant cell.

<table>
<thead>
<tr>
<th>( w_{t+1}/(1+r_{t+1}) )</th>
<th>( w_t )</th>
<th>( X_t )</th>
<th>( z_t^* )</th>
<th>( g_t )</th>
<th>( r )</th>
<th>( B )</th>
<th>( m )</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_t )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( A_t )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2 identifies model parameters that affect importantly the set of skilled workers \( A_t \).\(^{10}\) Among them, tuition cost \( z_t^* \) is at the centre of current policy debates in the USA as access to higher education is increasingly threatened by tuition growth.\(^{11}\) Also, parameter \( m \) stands out since together with \( w_t \) it represents lost earnings while studying and captures therefore the opportunity cost of higher education. As parameter \( m \) is inversely related to the ending age of compulsory schooling it is determined largely, though not exclusively, by institutions. One should expect

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\(^9\)Eicher (1996) models also a partition of the labor force between skilled and unskilled workers but it is individuals who make their own occupation choice based on the respective career paths as skilled or unskilled.

\(^{10}\)The allocation of individuals at generation \( t \) between the groups of skilled and low-skilled workers does not depend on the intensity of altruism \( \alpha_t \). Likewise, the stock of human capital \( \text{H}_t \) is independent of the altruism parameter. Thus, in our model the intensity of altruism does not affect growth, as long as \( \alpha_t > 0 \). This result is in contrast to the result obtained in dynastic models like that of Armellini and Basu (2009).

\(^{11}\)This standpoint was emphasized by President Barack Obama in a recent speech at the University of Michigan: “We are putting colleges on notice: you can’t assume that you’ll just jack up tuition every single year” (International Herald Tribune; March 10-11, 2012).
therefore large cross-country differences in the opportunity cost of higher education.\footnote{For example, the ending age of compulsory schooling is 18 for Chile and the Netherlands, 15 for Israel and 14 for Turkey (see the OECD web site for updates).}

Table 2 also reveals some insights regarding how globalization affects the process of skill formation. To that end consider a counterfactual where full capital mobility is implemented once an autarkic equilibrium is in place. Upon free capital mobility, physical capital will flow from the low-return to the high-return country. If the domestic marginal return decreases (increases) to the world interest rate the economy will experience an expansion (a reduction) of its skilled workforce. Summarizing:

**Corollary 1:** Under the above assumptions, we obtain in equilibrium that: a higher wage-rental ratio \( \frac{w_{t+1}}{1+r_{t+1}} \) at date \( t+1 \) expands the set of skilled agents at that date, while a lower wage-rental ratio enlarges the set of low-skilled labor.\footnote{Some empirical studies have shown that globalization has been a small contributor to growing wage inequalities and to the size of the unskilled workforce in trading nations (see, e.g. Greenaway and Nelson, 2000; Winchester, 2008). Our result in Corollary 1 proposes a different explanation to the size of the low-skilled workforce: the decision is made by altruistic rational parents who give significant weight to the ability of their child, the family background and the foregone income due to the time spent acquiring higher education. They then decide whether to invest in their child's higher education or, perhaps, let him/her start working right after compulsory schooling and, hence, become a low-skilled worker.}

Note that the factor price ratio in Corollary 1 is determined by the parameters of the production function.

**Role of Government Budget Balance**

The signs of Table 2 are obtained with incomplete consideration regarding the notion of equilibrium. For example, \( X_t \) and \( g_t \) enter threshold \( \Lambda_t \) directly with no acknowledgement of budget balance. It is therefore crucial to be more precise about the response of threshold parameter \( \Lambda_t \) to public funding, noting that the government budget must be balanced in equilibrium.
Since in equilibrium the educational policy must be feasible (hence, condition (8) holds), we observe that reducing the public funding of higher education, results in crowding in of private funds, and as a result lowers the total number of students in higher education. This can be verified from the variation in the "cut off" point $\Lambda_i$: reducing $g_i$ implies by (8) that $X_i$ increases. Similarly, raising public funding will crowd out private funding and expand higher education. Let us consider the extreme scenario of full public funding of higher education, namely, $\hat{g}_i = z_i^*$ for all $t$. In this case, $z_i = 0$ and from (20) we obtain that all individuals $\omega$ invest in higher education given that condition (19) holds at all dates. Exogenous factor prices thus play an important role in the formation of types of workers and to guarantee that skilled individuals exist in each generation, we assume:

(A2) Given the exogenous wages and interest rates, the economy's parameters $m$ and $B$, condition (19) holds at all dates $t$, $t=0, 1, 2, .....$

Formally, it is important to obtain the response of $(z_i^* - g_i)/X_i^*$ to the public subsidy. The left-hand side of (8) is simply $\tau w_i H_t$, a useful shorthand expression for government tax revenues. Denote by $\gamma_t$, $0 \leq \gamma_t \leq 1$, the fraction of government revenues at date $t$ allocated to compulsory schooling. Then:

(21) $X_t = \gamma_t \tau w_t H_t$

(22) $g_t \mu(A_t) = (1 - \gamma_t) \tau w_t H_t$

With $\gamma_t = 1$, public funding of higher education is zero ($g_t = 0$) and tertiary education is fully privately financed. With $g_t = z_t^*$, higher education is fully publicly financed. Using the above equations:
To obtain the effect of higher expenditure in compulsory schooling in equilibrium, we derive from this expression (using some earlier conditions):

\[
\frac{(z^*_t - g_t)}{X^*_t} = \frac{z^*_t - (1 - \gamma_t) \tau w_t H_t / \mu(A_t)}{(\tau \gamma_t w_t H_t)^{\xi}}
\]

In contrast to the straightforward comparative static results of Table 2, (24) gives the response to an increase in the share of expenditure in compulsory education knowing that the increase is achieved at the cost of higher education. Namely, the partial derivative is positive as long as \( X_t / \mu(A_t)(z^*_t - g_t) > \xi \). This condition holds if: (i) per-student public expenditure on compulsory schooling \( X_t \) is usually higher than per-student private expenditure on higher education \( (z^*_t - g_t) \); (ii) \( \mu(A_t) < 1 \) (less than 0.5 in many economies) and \( \xi < 1 \). It follows that we then obtain a positive effect on the threshold parameter \( \Lambda_t \). \textit{Vice versa}, an increase in funding of higher education (a decrease in \( \gamma_t \)) leads to a decrease in \( \Lambda_t \) and to an increase in \( \mu(A_t) \). Increasing or decreasing \( \gamma_t \) does not affect the set \( A_t \) when \( \mu(A_t) = X_t / (z^*_t - g_t) \xi \).

\textbf{Economy’s Human Capital}

Finally, another important question arises: is it always true that an expansion of the set of skilled workers leads to a higher aggregate stock of human capital that is available for production activities? It all depends on the causes of this expansion since variables and parameters of the model have a different status. For example, \( w_t, m, \tau, g_t, X_t, B, \xi \) interfere directly with both sides of government budget balance while tuition fee \( z^* \) and wage-rental ratio \( (w_{ts} / (1 + r_{ts})) \) only affect the expenditure.
side via $\mu(A_i)$. The next proposition discusses only a few of these predetermined variables and to fix ideas let us make the following assumption:

(A3) \[ B > 1+m \] holds.

In assumption (A3) $m$ measures the time a low-skilled worker’s human capital is used in production. $(B-1)$ measures the increased qualification this worker would get if he were to attend college instead. Hence, $B-1-m>0$ guarantees that the individual’s human capital made available for productive activities is higher if he decides to attend higher education rather than to remain low skilled.\(^ {14} \)

**Proposition 2:** Under assumption (A3), output declines at the current date $t$ but expands in all subsequent periods $t+k$, $k \geq 1$, in each of the following two cases taking place at date $t$: (a) An unexpected increase in the wage-rental ratio; (b) A technological progress in the education sector (higher $B$ or higher $\xi$).

The proof is based on the result of Corollary 1 and on the next lemma.

**Lemma 1:** Under assumption (A3), expanding the set $A_i$ at date $t$ results in a lower $H_t$ but a higher $H_{t+k}$ for all $k \geq 1$.

The initial decline in productive human capital and output is due to a lower labor market participation by the youths more of whom will go to college. Thus in Lemma 1 we observe an expansion of the set $A_i$ at date $t$ and lower $H_t$. Suppose the cause of the increase in $A_i$ is a technological improvement in primary education (a higher $\xi$). Some individuals who were planning initially to be low skilled now decide to study longer and therefore leave the ranks of low-skilled workers. The stock of

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\(^ {14} \)Alternatively parameter $B$ represents also the education wage gap between a skilled worker with a college degree relative to that of a low-skilled worker with high school and less. Using information on $m$, a testable hypothesis is to verify whether the education wage gap of any country exceeds the country-specific lower bound $(1+m)$. See Hotchkiss and Shiferaw (2011) and the references therein for measurement and estimation methodologies of the education wage gap.
human capital available for production $H_t$ decreases in period $t$ and the economy that observes also an outflow of physical capital faces a decline in output at the current date $t$ as in Proposition 2.

4. The Value of Public Funding of Higher Education

Having described the sets of low-skilled and high-skilled workers, we now turn to the main issue of our study, namely what is the role of a government in enhancing higher education? We shall investigate the conditions under which increasing public funding will enhance the formation of skilled workers and the resulting effects on economic growth and efficiency. This section will begin with the impact of public funding of higher education on the aggregate stock of human capital. We shall analyze the impact at date $t$ first and then focus on the dynamic process and efficiency issues.

**Impact Effects**

On the expenditure side of its balance sheet the government faces public expenditure in higher education equal to $\mu(A_t)g_r$. Enrollment in higher education is costly and requires a net payment from private sources equal to $\mu(A_t)(z_r^* - g_r)$. Therefore, $(1 - g_r/z_r^*)$ represents the fixed *share of private investment* in total expenditure on higher education, not only for each individual but also for society as a whole. A decision by schools to charge a higher tuition fee $z_r^*$ increases this share while a larger public support will decrease it. Data reveal that in tertiary education the proportion of costs funded privately varies widely across our sample of countries. In Chile and Korea for example, public funding represents only a small part of investments in tertiary education. In contrast, approximately 73 percent of expenditure on higher education is public in the Netherlands (OECD, 2009). Given our framework, we obtain the next result:
Proposition 3: Assume that condition $X_t / \mu(A_t)(z^* - g_t) > \xi$ holds and that initially at some period $t$ we have $g_t = 0$. An increase in the public funding of higher education to some positive level $g_t > 0$ leads in equilibrium to: (i) a larger set of skilled agents at date $t+1$; (ii) a lower total expenditure on education at date $t$, and (iii) a lower stock of human capital $H_t$ used in production at date $t$.

The proof of this Proposition demonstrates how this change in public funding yields an increased enrolment in colleges due to "lower cut off level $\Lambda_t". In other words, given the distribution of $Z_{t+1}(\omega)$, then using the expression for $\Lambda_t$ we can derive the set of agents who enter higher education following this change. The condition in this Proposition requires that the ratio of average expenditure on basic schooling to per-student spending on higher education is bounded **from below** by $\xi < 1$. The fact that $H_t$ decreases in period $t$, due to lower college participation of the younger generation, corroborates the finding of Proposition 2 and extends the result to a more complex environment. Following Proposition 3 we also derive:

**Corollary 2:** In equilibrium with balanced budget, the opportunity cost of increasing resources in favour of higher education is larger than unity.

The reason is that some unskilled workers who previously contributed to tax revenues now become users of higher education subsidies to become skilled.

**Dynamic Analysis**

Now let us consider the effect of increasing public funding of higher education to enhance the formation of skilled labor (along a feasible education program). Consider the case where the government proposes two policies: either ‘no public funding’, i.e. $g_t = 0$, or the long-run policy $\{\bar{g}_t\}_{t=0}^\infty$, which guarantees at each date $t$
the **per-student funding** at a positive level $\bar{g}_t$. At date $t$, let the set of families who opt for a ‘skilled child’ under the ‘no funding’ policy be defined by:

\begin{equation}
A^0_t = \{ \omega | Z_{t+1}^0(\omega) \geq \frac{1}{(1-\tau)(B-1)} \left[ \frac{1}{w_{t+1}^{i+1}} - \frac{m}{w_t^i} \right] \frac{z^*_t}{\tau w_t^i H_t^{1/P}} = \Lambda^0_t \}
\end{equation}

Let us denote the set of families at period $t$ who opt for a ‘skilled child’ under the 'per-student public funding $\bar{g}_t,$' policy by:

\begin{equation}
\overline{A}_t = \{ \omega | Z_{t+1}(\omega) \geq \frac{1}{(1-\tau)(B-1)} \left[ \frac{1}{w_{t+1}^{i+1}} - \frac{m}{w_t^i} \right] \frac{z^*_t - \bar{g}_t}{\bar{X}_t^{1/P}} = \overline{\Lambda}_t \}
\end{equation}

Reducing the private cost of higher education will expand the set of skilled labor, namely, we have: $\overline{\Lambda}_t < \Lambda^0_t$. We shall make in the following proposition an assumption regarding the ‘sensitivity’ of the set of skilled labor to changes in the threshold level $\Lambda_t$. Let us rewrite the aggregate human capital of generation $t+1$:

\begin{equation}
\overline{H}_{t+1} = \int_{h_{t+1}(\omega)d\mu(\omega)} X_t^{\frac{P}{1-P}} \left[ B \int_{A_t} Z_{t+1}^0(\omega)d\mu(\omega) + \int_{A_t} Z_{t+1}(\omega)d\mu(\omega) \right]
\end{equation}

Does a certain level of public funding of higher education enhance human capital formation, and hence growth in our economy? The literature has some support for this claim (see, e.g., Bassanini and Scarpenta, 2001; Caucutt and Kumar, 2003; Blankenau, 2005; Arcalean and Schiopu, 2010). We show that in our framework such result depends on certain parameter values:

**Proposition 4:** Given the education tax rate $\tau$, assume that initially in equilibrium there is no government intervention in financing higher education. Introducing public funding of higher education in equilibrium at the level $\{\bar{g}_t\}_{t=0}^\infty$ varies the corresponding threshold levels from $\{\Lambda^0_t\}$ to $\{\overline{\Lambda}_t\}$. Define:

\begin{equation}
\overline{\Lambda}_t = \Lambda^0_t (1-d_t), \text{ for } t=1,2,\ldots.
\end{equation}
If \( d_t \leq \bar{g}_t / z_t^* \) holds for all \( t \), then the introduction of such public funding policy increases the stock of human capital at all dates; namely, \( H_t^0 < \bar{H}_t \) holds for \( t \geq 1 \).

Recall that \( \bar{g}_t / z_t^* \) is the share of the public funding in the per-student cost of higher education \( z_t^* \). Thus, if the sensitivity of the threshold levels to variations in the funding level is not ‘too high’, hence the resulting expansion of the set of skilled agents \( A_t \) is not ‘too rapid’, we obtain that higher public funding will enhance the creation of human capital. This condition depends basically on the ‘smoothness’ of the human capital distribution in equilibrium and in the density function of the random ability. In Proposition 4 condition (28) makes an assumption about the elasticity of the threshold levels for different levels of public funding. In contrast, the condition assumed in Proposition 3 compares the per-student investment in compulsory schooling with the average cost of higher education at some given date.

It can be verified from condition (20) that comparing these two equilibria the assumptions in this Proposition imply that: the corresponding per-capita public spending on compulsory education levels from \( \{X_t^0\} \) to \( \{ar{X}_t\} \) satisfy the condition \( \bar{X}_t \leq X_t^0 \) for \( t=1,2,.. \). Note that the tax rates are the same in both equilibria but not necessarily the tax revenues. Since (8) holds in equilibrium introducing subsidies to higher education does not necessarily guarantee the above conditions.

**The Possibility of Inefficiency of Public Funding**

Proposition 4 has implications for economic growth. The human capital accumulation resulting from the public funding of higher education is expected to increase domestic marginal returns to physical capital and, hence, generate a foreign inflow of physical capital. The increase in both primary inputs will increase output.
But does this outcome justify the diversion of public funds to finance higher education?

The answer depends on cost-benefit considerations: the relevant variable here is the net value of labor at the current date. Namely, it is the total additional income generated from this investment: The increase in labor income of generation $t$ minus the public expenditure at date $t$ on higher education. The reason is that, intergenerational transfers being given at the outset of each period, the working population’s only source of income is from labor.

To substantiate the assertion that society as a whole is not always better off when some public funds are used to finance higher education, consider the competitive equilibrium from some initial conditions and a given feasible education policy $\{(X_t, g_t)\}$. Under a given allocation of the education budget we define the net value of labor at date $t$, denoted by $W_t(X_t, g_t)$, as follows:

$$W_t(X_t, g_t) = [mw_t(1+r_{w_t}) + w_{r_{t+1}}] \int \bar{\theta}_t(\omega)h_t^c(\omega)X_t^c d\mu(\omega)$$

$$+ Bw_{r_{t+1}} \int \bar{\theta}_t(\omega)h_t^c(\omega)X_t^c d\mu(\omega) - g_t\mu(A_t)$$

Clearly, one can define this value in various meaningful ways. The assumption here is that the government cannot vary easily the allocation of the education resources in the short run, hence $\{(X_t, g_t)\}$ is given. We also ignore the cost of compulsory schooling since it cannot be changed. For some initial conditions at $t=0$, we say that a feasible education policy $\{(X_t^*, g_t^*)\}$ dominates another feasible education policy $\{(X_t, g_t)\}$ if at any date $t$, switching from $(X_t, g_t)$ to $(X_t^*, g_t^*)$ is desirable in the following sense:

(a) $W_t(X_t^*, g_t^*) > W_t(X_t, g_t)$.
At each date $k$, if the government has to choose between these two education policies, then $(X_k^*, g_k^*)$ will have a higher net value of labor, that is,

$$W_k(X_k^*, g_k^*) > W_k(X_k, g_k).$$

This definition of net value of labor implies that the policy $\{(X_t^*, g_t^*)\}$ generates more net aggregate income for each generation. It is favored by each generation which compares options under the current distribution of human capital at the outset of the period. Let us now compare two simple cases: the no-public funding policy, denoted by $\{(X_t^0, g_t^0 = 0)\}$ and the full-public funding policy, denoted by $\{(X_t^*, g_t = z_t^*)\}$.

**Proposition 5:** Assume that the following two conditions hold:

1. $X_t^0 / z_t^* > \xi$, for all dates $t$, and
2. $B^{\frac{1}{\gamma}}[1 - z_t^*/\tau\eta; H_t^0] \leq 1$, for all dates $t$.

Then, the no-public funding policy dominates the full-public funding policy.

Under the above conditions the cases where the government does not allocate public funds to higher education may be “better” from the point of view of economic efficiency than the fully-funded cases (which we observe in many European countries). Though condition (29) is tighter than what has been assumed in Proposition 4, it remains a mild assumption. Condition (30) requires that $B$ should not be 'too large' and/or the per-student cost of higher education is not too 'small' compared to the average per-student public education expenditure. Also, when $\xi$ is close to 1 and $B$ is not ‘too high’ it helps condition (30) to be satisfied. The result in this proposition may be in contradiction to the case studied in Proposition 4 where we move from 'no funding' to a "small" level of funding of tertiary education: in this case it might be 'more efficient' to provide low-level of funding than none!
5. Political Equilibrium

So far we assumed that the allocation of the public education funds (hence $\gamma_t$) within the educational system is exogenously given. This assumption is questionable since the allocation of government revenues between these two types of education stages is likely to vary with changes in the educational technology of early education vs. college education, market conditions at home and abroad, etc. Table 3 that compares the shares of public expenditure on tertiary education (as a percentage of total public expenditure on education) reveals a large diversity between countries: the largest share $\gamma_t$ is observed for Turkey; Korea and Chile have the smallest shares. Clearly the latter countries rely heavily on private funding to finance higher education.

Table 3: Public Expenditure on Tertiary Education $^{a,b}$

<table>
<thead>
<tr>
<th>OECD Countries</th>
<th>(1 - $\gamma_t$)</th>
<th>Partner Countries</th>
<th>(1 - $\gamma_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>16.84</td>
<td>Brazil</td>
<td>16.67</td>
</tr>
<tr>
<td>Korea</td>
<td>14.67</td>
<td>Chile</td>
<td>15.06</td>
</tr>
<tr>
<td>Mexico</td>
<td>17.27</td>
<td>Estonia</td>
<td>19.44</td>
</tr>
<tr>
<td>Netherlands</td>
<td>27.50</td>
<td>Israel</td>
<td>16.79</td>
</tr>
<tr>
<td>Portugal</td>
<td>19.46</td>
<td>Russian Fed</td>
<td>22.14</td>
</tr>
<tr>
<td>Turkey</td>
<td>31.03</td>
<td>Slovenia</td>
<td>21.71</td>
</tr>
</tbody>
</table>

Notes: (a) As a percentage of total public expenditure on education; (b) In 2007.

Source: Authors’ own calculations based on OECD (2009, Table B4.1)

In economies with heterogeneous agents, the choice of an ‘optimal’ $\gamma_t$ can be determined via the outcome of some political process at each date. It is possible to establish a mapping between the set of heterogeneous agents, given their preferences regarding education, and an ‘optimal’ education policy determined by majority voting. Economies at different stages of development, with a different composition of the labor force, are expected to reach different political equilibria regarding this educational budget allocation.

Preferences of Agents
Let us express individual income as a function of $\gamma_t$ by substituting away $x_t$ and $g_t, \mu(A_t)$. Making use of (5), (6), (16) and (17) income of agent $\omega$ who is either a low-skilled or a skilled offspring has the following reduced form:

$$y'_{t+1}(\omega) = \left( \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \right) \left( 1 + r_{t+1} \right) \left\{ (1-\tau) (w_{t+1} + mw_t (1+r_{t+1})) \frac{Z_{t+1}(\omega) \gamma_t^\tau \xi^\tau w_t^\tau H_t^\tau + y_t(\omega)}{(1+r_{t+1})} \right\}$$

$$y''_{t+1}(\omega) = \left( \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \right) \left( 1 + r_{t+1} \right) \left\{ \frac{(1-\tau) w_{t+1}}{(1+r_{t+1})} B Z_{t+1}(\omega) \gamma_t^\tau \xi^\tau w_t^\tau H_t^\tau + y_t(\omega) \right\} - z^* + \left( \frac{1-\gamma_t}{\mu(A_t)} \right) \tau w_t H_t$$

Given the parameters at each date $t$ including $H_t$ and $y_t(\omega)$, both expressions for next generation’s income are strictly concave function of $\gamma_t \in [0,1]$. This implies that the optimal choice $y_t(\omega)$ of each agent is unique.

However, a high $\gamma_t(\omega)$ might not be realistic since implementing a university system requires important resources. High education involves large fixed costs associated to setting up the institutions and to offering a minimal curriculum. There is thus an upper bound for $\gamma_t(\omega)$ and, therefore we compare two situations: each individual votes either for no public funding, i.e., $g_t = 0$, or for public funding at level $g_t = \bar{g}_t$. The choice will be determined by comparing the income of his/her offspring under these two policies, namely: given $Z_{t+1}(\omega)$ we compare $y'_{t+1}(\omega)$ under $g_t = 0$ to $y''_{t+1}(\omega)$ under $g_t = \bar{g}_t$. Denote by $\bar{\gamma}_t$ the fraction of the education budget assigned to compulsory schooling when higher education is publicly funded with $g_t = \bar{g}_t$. The condition that determines voting in support of $g_t = \bar{g}_t$ is given by:

$$\frac{(1-\tau) w_{t+1}}{1+r_{t+1}} B Z_{t+1}(\omega) [\tau w_t H_t \bar{\gamma}_t^\tau] \xi^\tau = z^* + \bar{g}_t + y_t(\omega) \geq$$

$$\left[ \frac{1+r_{t+1}}{1-\tau} \right]^{-\frac{1}{\tau}} [w_{t+1} + mw_t (1+r_{t+1}) Z_{t+1}(\omega) [\tau w_t H_t] \xi^\tau + y_t(\omega)]$$

Rearranging terms implies:
\[ Z_{t+1}(\omega) \geq v_t, \]

where

\[ v_t = \frac{(z_t^* - \bar{g}_t)(\tau w_t H_t)^{\xi}}{(1 - r)(B(\bar{g}_t)^{\xi} - 1)\frac{w_{t+1}}{1 + r_{t+1}} - mw_t}^{-1}. \]

(31)

Like \( \Lambda \) in Proposition 1, \( v_t \) in (31) is another threshold that partitions the distribution of endowments, namely between those who favour public funding for higher education at level \( g = \bar{g}_t \) versus those in favour of the alternative policy \( g = 0 \).

Namely, all voters with endowment \( Z_{t+1}(\omega) \geq v_t \) will vote in favour of public funding, all others will vote against.

Threshold \( v_t \) is another channel through which international market conditions affect the education system. For example, a higher wage/rental ratio at the next period (resulting from globalization and liberalization of capital markets) implies a larger group of individuals who support \( g = \bar{g}_t \). Also partition parameter \( v_t \) responds negatively to the changes in the following parameters: (i) In a society endowed with a larger stock of human capital \( H_t \), more people support larger public resources be allocated to higher education; (ii) As public education expenditures \( (\tau w_t H_t) \) increase more individuals support an increase in resources for higher education; (iii) A lower value of \( m \) or larger value of \( \xi \) imply more support for the policy \( g = \bar{g}_t \). Again, it is notable that \( v_t \) does not depend on the intensity of altruism.

Further insights into the voting behaviour of individuals in generation \( t \) can be gained by comparing the position of partition parameters in the distribution of endowments. They are summarized in the next two claims:

**Claim 1:** \( v_t > \bar{\Lambda}_t \).

**Claim 2:** \( \bar{\Lambda}_t < \Lambda_t^0 \) holds for all \( t \).
The proofs of the two claims are included in the appendix. The following corollaries follow directly from Claim 1 and Claim 2:

**Corollary 3:** Some of the agents who voted against instituting public funding for higher education will invest in higher education when public funding is provided.

**Corollary 4:** Some of the households who did not invest in higher education under the no-public funding regime will invest in higher education with public funding.

**Majority Voting**

In order to reach a political equilibrium, what matters is to know the relative position of the median voter \( M \) in the distribution of initial endowments. Let \( Z_{t+1}(M) = \tilde{h}(M) h(M) \) be his/her initial endowment. Hence:

**Proposition 6:** When the allocation of education resources is determined by a political equilibrium, applying the Median Voter theorem implies that public funding is approved, i.e., \( g_t = g_r \) if and only if \( Z_{t+1}(M) \geq \nu_t \). Thus the shape of the distribution of endowments in generation \( t \) matters for the determination of the equilibrium.

We obtain that in a society with a majority of low-skilled workers with low endowments the median voter is in favour of not allocating public resources to college education (Blankenau et al., 2007a). This result is clear in a small open economy: parents of generation \( t \) who are aware that their child is becoming a low-skilled worker will not benefit from supporting public funding for higher education. They perceive public funds assigned for higher education as a net transfer of government resources from them to individuals who shall mostly have high income in the future.\(^{15}\)

\(^{15}\)If, in addition, the conditions of Proposition 5 are met, then the choice of low-skilled voters is desirable as well. In richer economies with a majority of skilled workers the allocation of resources depends on the shape of the distribution of endowments of individuals in that generation. If the condition of Proposition 6 is met, the government allocates public resources to higher education and the predictions of Propositions 3 and 4 are applicable in this context.
6. Concluding Remarks

Is it always desirable that public funds be used to finance higher education? It is the main question that has been raised by this paper. The answer may depend on the underlying features of the economy, such as cost and productivity of the higher education system and other parameters describing the process of skill formation. In some cases we demonstrate that such public funding will enhance the formation of human capital and thus promote economic growth. We also derive conditions under which public financing of higher education is inefficient. In other words, in some small open economies refraining from using public resources for higher education can 'dominate' the regime in which the government fully funds higher education. Thus, using public funds to send 'low quality' students to college may be inefficient since the government has better alternatives like using these resources to improve the compulsory schooling system (which is benefiting all students).

The tremendous expansion of globalization in the last three decades has affected small open economies very significantly and its impact on education policy and skill formation is a significant topic. The relevant theoretical literature (see, e.g., De Fraja, 2002, and many others) has studied educational policies mostly within closed economies, while our aim was to promote our understanding of these relationships in small open economies. We explore the role of international capital mobility in affecting education choices as well as governmental decisions related to public funding. Our results may be relevant to certain small open economies but not to others. Some of the conditions we have assumed are related to the productivity of advanced education, the cost of attaining skills, the prices of international factors and the importance of the initial distribution of human capital among countries.
The framework we have applied has several important features, some of which contribute to our results in a significant way. For example, we take into account parental altruism and the opportunity cost of attending higher education. It is not clear to us how robust the results are when we dispose of such assumptions. However, we feel comfortable with such assumptions since they add realism to the analysis. Though we have allocated individuals in this economy to groups of skilled and low-skilled workers we abstained from studying the effects of international factors on income inequality. This important issue should be considered in future research. In a different framework, Viaene and Zilcha (2002) have examined the effect of international factors on income distribution in equilibrium.

7. Appendix

**Proof of Proposition 1:** Consider the case where the child is skilled. Substitute first order conditions in (11) and solve for $b_t(\omega).$ Making use of (17) we are able to solve for $y_{t+1}^s(\omega)$. Repeat the same steps for the case where the same child is low skilled to derive $y_{t+1}^l(\omega).$ Hence,

\[
y_{t+1}^s(\omega) \geq y_{t+1}^l(\omega) \iff U_t^s(\omega) \geq U_t^l(\omega)
\]

implies:

\[
(1 - \tau)\tilde{B} \tilde{\theta}_t(\omega) \hat{h}_t(\omega)^\nu X_t^z w_{t+1} - z_t \geq (1 - \tau)\tilde{\theta}_t(\omega)\hat{h}_t(\omega)^\nu X_t^z \left[w_{t+1} + (1 + r_{t+1})mw_t\right]
\]

Note that this inequality holds only if condition (19) holds. Moreover, it is easy to verify that when (19) holds the set of skilled individuals is given by (20).

**Proof of Corollary 1:** Let us rewrite the condition that defines the set of individuals $\omega \in G_t$ who choose to assume higher education:

\[
Z_{t+1}(\omega) = \tilde{\theta}_t(\omega)\hat{h}_t(\omega)^\nu \geq \frac{1}{1 - \tau} \left[\frac{1}{w_{t+1}} \right] \frac{\lambda_t}{X_t^z} = \Lambda_t
\]

Assume that at date $t$ we have a higher interest rate $(1 + r_{t+1});$ this implies a lower wage-rental ratio $w_{t+1}/(1 + r_{t+1}).$ As a result, noting that condition (19) remains valid and examining the definition of $A_t,$ we find that the value of $\Lambda_t$ increases since the
private investment $z_t$ and public investment in compulsory schooling $X_t^s$ remain unchanged. Hence the set of skilled agents $A_t$ shrinks. Similarly, lowering the rate of interest will lower $\Lambda_t$, hence expanding the set of skilled workers $A_t$. ■

**Proof of Lemma 1:** Recall the definition of the stock of human capital at date $t$:

$$H_t = \int h_t(\omega) d\mu(\omega) + m \int h_{t+1}(\omega) d\mu(\omega)$$

As $A_t$ increases, the first term in this expression remains unchanged while the second decreases. Hence $H_t$ drops. Consider now later periods:

$$H_{t+1} = \int h_{t+1}(\omega) d\mu(\omega) + m \int h_{t+2}(\omega) d\mu(\omega)$$

There are two effects. First, low-skilled workers join the skilled workforce: $A_t$ increases but $\sim A_t$ decreases by the same number. Second, less low-skilled workers induce their child to be low-skilled workers as well, this at date $t+1$ through the endowment condition: $\sim A_{t+1}$ decreases (hence $A_{t+1}$ expands). Consider now two situations and denote the corresponding sets of skilled workers by: $A_1^0$ and $A_0^0$ with $\mu(A_1^0) > \mu(A_0^0)$. Since we transfer unskilled workers to the skilled labor force we obtain that $\int h_{t+1}(\omega) d\mu(\omega)$ increases. On the other hand, since $A_{t+1}$ expands we obtain that $\int h_{t+2}(\omega) d\mu(\omega)$ increases. Let us write:

$$H_{t+1}^0 = \int h_{t+1}^0(\omega) d\mu(\omega) + m \int h_{t+2}^0(\omega) d\mu(\omega)$$

Let us denote by $\Delta_t = [\sim A_1^0] / [\sim A_0^0]$, then for any $\omega \in \Delta_{t+1}$ we have by our assumptions: $h_{t+1}^1(\omega) \geq Bh_{t+1}^0(\omega)$, hence

$$\int h_{t+1}^1(\omega) d\mu(\omega) \geq B \int h_{t+1}^0(\omega) d\mu(\omega) \geq (1 + m) \int h_{t+1}^0(\omega) d\mu(\omega)$$

This implies that $H_{t+1}^1 - H_{t+1}^0 \geq m \int h_{t+1}^0(\omega) d\mu(\omega)$. This process can be continued for all coming dates since we obtained that $A_{t+1}^0$ also expands. Thus our claim is proved.
Proof of Proposition 3: For some $t$ assume that $g_t$ increases. Let us rewrite (8) as follows:

\[(8') \quad \tau w_t \left[ \int h_t(\omega) d\mu(\omega) + mX_t^\xi \tilde{\theta} \int_{-A_t} h_t(\omega)^\nu d\mu(\omega) \right] = X_t + g_t \mu(A_t) \]

since $\tilde{\theta}_t(\omega)$ are i.i.d. Any increase in $g_t$ decreases parameter $\gamma_t$. By (20), as $g_t$ expands, $z_t/X_t^\xi$ decreases, which clearly implies a decrease in $X_t$. Since $\Lambda_t$ declines we obtain that the set $A_t$ expands. From (8') we see that $H_t$ decreases as well even though $g_t \mu(A_t)$ increases. Thus, total expenditures on education decrease. The drop in $X_t$ is larger than the initial increase in $g_t$: the marginal rate of substitution between $X_t$ and $g_t$ is therefore larger than 1 in absolute value. ■

Proof of Proposition 4: Write: $z^*_t = z^0_t$ and hence, $\bar{z}_t = z^*_t - \bar{g}_t$. Thus:

\[
\frac{z^*_t}{(X_t)^\xi} = \frac{z^0_t}{(X_t)^\xi} - \frac{\bar{g}_t}{(X_t)^\xi} = \frac{z^*_t}{(X_t)^\xi}(1 - d_t)
\]

We obtain from this equation,

\[
\frac{z^*_t}{(X_t)^\xi} \left[ \frac{(X_t)^\xi}{X_t} - 1 + d_t \right] = \frac{\bar{g}_t}{(X_t)^\xi}
\]

which yields:

\[
\frac{(X_t)^\xi}{X_t} = \frac{1 - d_t}{1 - \bar{g}_t/z_t^*}
\]

Now, let us define $Q(\bar{g}) = B \int_A h_t^\nu(\omega) d\mu(\omega) + \int_{-A} h_t^\nu(\omega) d\mu(\omega)$ and write the expressions for the ratio of generational aggregate human capital:

\[
\frac{\bar{H}_{t+1}}{H_{t+1}} = \frac{(X_t)^\xi}{X_t} \frac{\partial Q(\bar{g}_t)}{\theta (0)} = \frac{1 - d_t}{1 - \bar{g}_t/z_t} \frac{Q(\bar{g}_t)}{Q(0)}
\]

Since $\bar{\Lambda}_t < \Lambda^0_t$ the set of skilled workers with the subsidy contains (strictly) the set under 0 subsidy, namely: $A_t \subset \bar{\Lambda}_t$, hence $Q(\bar{g}_t) > Q(0)$. Thus, by our assumption, we obtain that $H^0_{t+1} < \bar{H}_{t+1}$ for all $t$. ■

Proof of Proposition 5: Suppose that we switch from zero-public funding to full-public funding at date $t$. Comparing the net labor income in these two cases, the Proposition requires that:
\[ W_t(X_t^0, 0) = [mw_t + w_{t+1}] \int_{A} \tilde{\theta}_t(\omega)h_t^{\psi}(\omega)X_t^0 + Bw_{t+1} \int_{A} \tilde{\theta}_t(\omega)h_t^{\psi}(\omega)X_t^0 > \]

(E.1)

\[ W_t(X_t^*, 0) = Bw_{t+1} \int_{A} \tilde{\theta}_t(\omega)h_t^{\psi}(\omega)X_t^* - z_t^* \]

But the right hand side of (E.1) can be rewritten as follows:

\[ W_t(X_t^*, 0) = Bw_{t+1} \hat{X}_t^* \int_{A} Z_{t+1}(\omega) + w_{t+1} \hat{X}_t^* \int_{A} Z_{t+1}(\omega) \]

Thus, the inequality in (E.1) holds if the following inequality holds:

\[ mw_t(1+r_{t+1})(X_t^0)^{\xi} \int_{A} Z_{t+1}(\omega) + w_{t+1}[(X_t^0)^{-\xi} - B\hat{X}_t^\xi] \int_{A} Z_{t+1}(\omega) > Bw_{t+1}[\hat{X}_t^\xi - (X_t^0)^{-\xi}] \int_{A} Z_{t+1}(\omega) - z_t^* \]

A sufficient condition for this inequality to be satisfied is: \((X_t^0)^{\xi} \geq B\hat{X}_t^\xi\). This can be rewritten as: \(X_t^0 \geq \frac{B}{\xi} \hat{X}_t^\xi\). Rewriting this inequality:

(E.2) \[ \tau w_tH_t^0 \geq B^{\frac{1}{\xi}} [\tau w_t\hat{H}_t - z_t^*] \]

Using Proposition 3 we obtain that by increasing public funding from \(g_t^0 = 0\) to \(\hat{g}_t = z_t^*\) the period \(t\) stock of human capital will decline; namely, that \(\hat{H}_t < H_t^0\). Now, from (E.2) we obtain:

\[ 1 \geq B^{\frac{1}{\xi}} \frac{\hat{H}_t}{H_t^0} - \frac{z_t^*}{\tau w_tH_t^0} \]

Thus, we attain that condition (30) of the Proposition implies condition (E.2). Now, in each date \(k > t\), given the initial distribution of human capital, a choice between these two public funding regimes requires the same type of comparison as we did for date \(t\). Hence, when the conditions required in this Proposition hold at date \(k\) we obtain the same outcome. ■

**Proof of Claim 1:** Let us rewrite the expression for \(v_t\) as follows:

(E.3) \[ v_t = \frac{(z_t^* - \bar{g}_t)[\tau w_tH_t^0]^{-\xi}}{(1-\tau)}[(B+(\bar{\gamma})^{\xi} - (\bar{\gamma})^{\xi})^{-1} \frac{w_{t+1}}{1+r_{t+1}} - (\bar{\gamma})^{\xi} mw_t]^{-1} \]

From (31) and (E.3) we see easily that \(v_t > \bar{\Lambda}_t\) holds if and only if:

\[ [(B-(\bar{\gamma})^{\xi})^{-1} \frac{w_{t+1}}{1+r_{t+1}} - (\bar{\gamma})^{\xi} mw_t] < (B-1) \frac{w_{t+1}}{1+r_{t+1}} - mw_t \]

which holds since \((\bar{\gamma})^{\xi} > 1\). ■
Proof of Claim 2: To prove this claim let us define: 

\[ h_t(y) = \frac{z_t^* - y}{(A - y)^\xi} \]

where the positive constant \( A \) is \( \tau w_i H_t \). By straightforward calculation we verify that \( h'(y) < 0 \) since

\[ \frac{z_t^* - y}{A - y} < 1 \text{ and } \xi < 1. \]

Thus:

\[ \frac{z_t^*}{(\tau w_i H_t)^\xi} > \frac{z_t^* - \bar{g}_t}{(\tau w_i H_t - \bar{g}_t)^\xi} > \frac{z_t^* - \bar{g}_t}{[\tau w_i H_t - \bar{g}_t, \mu(A)]^\xi} \]

8. References


Product of Two Continuous Random Variables”, *Computational Statistics and Data Analysis* 44(3), 451-464.


